

# Background constraints in the infinite tension limit of the heterotic string

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## Abstract

In this work we investigate the classical constraints imposed on the supergravity and super Yang-Mills backgrounds in the  $\alpha' \rightarrow 0$  limit of the heterotic string using the pure spinor formalism. Guided by the recently observed sectorization of the model, we show that all the ten-dimensional constraints are elegantly obtained from the single condition of nilpotency of the BRST charge.

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## 1 Introduction

About three years ago, Cachazo, He and Yuan (CHY) proposed a compact formula for computing tree-level amplitudes in both Yang-Mills and gravity theories [1]. There was an increasing interest then to find a string origin of those results given their known connection to string amplitudes at the low-energy limit.

Soon after that work, Mason and Skinner introduced the so-called ambitwistor string [2], which could be viewed as an  $\alpha' \rightarrow 0$  limit of the usual string and provided a clear derivation of the CHY formulae for  $D = 10$  Yang-Mills and NS-NS supergravity.

Taking advantage of the pure spinor formalism's manifest supersymmetry, Berkovits proposed its ambitwistor version in [3], which was explicitly shown in [4] to provide the supersymmetric version of the CHY amplitudes.

When extended to curved backgrounds, one would expect that consistency of the ambitwistor string should put the target space fields on-shell. In [5], Adamo *et al* demonstrated that the nonlinear equations of motion of the NS-NS background arise as anomalies of the worldsheet supersymmetry algebra. In the pure spinor case, Chandia and Vallilo investigated the type II background [6] and realized that Berkovits' original proposal for the infinite tension string was incomplete and had to be modified in order to obtain the usual background constraints coming from the pure spinor formalism. By performing a semi-classical analysis, they were able to reproduce the known results of [7] with the introduction of the extra condition of BRST-closedness of  $\mathcal{H}$ , a generalized particle-like Hamiltonian.

The ideas in [6] were further explored by one of the authors in [8] and it was shown that the new model, although still chiral, could be interpreted in terms of two sectors resembling the usual left and right-movers of the superstring. This construction was also extended to the heterotic case, providing a sensible description of the massless heterotic spectrum in this  $\alpha' \rightarrow 0$  limit. This was achieved by incorporating the observed sectorization in the heterotic BRST charge, which was then redefined to be

$$Q = \oint \{ \lambda^\alpha d_\alpha + \bar{c} T_+ - \bar{b} \bar{c} \partial \bar{c} \}, \quad (1.1)$$

where  $\lambda^\alpha$  is the pure spinor ghost,  $d_\alpha$  is the improved worldsheet realization of the superderivative

introduced in [6],  $(\bar{b}, \bar{c})$  are the reparametrization ghosts and  $T_+$  accounts for one of the sectorized energy-momentum-like tensors, which are defined in terms of  $\mathcal{H}$  and the full energy-momentum tensor  $T$  as

$$T_{\pm} \equiv \frac{1}{2}(T \pm \mathcal{H}). \quad (1.2)$$

As we show in the present work, the problem of finding the constraints on the heterotic background is somewhat more natural than in type II, in that  $\mathcal{H}$  enters the BRST charge  $Q$  itself, *cf.* (1.1), and the background constraints all come from the sole requirement that  $Q$  be nilpotent. In a general heterotic background, the action for the sectorized model and the generalized particle-like Hamiltonian will be cast as

$$S = \frac{1}{2\pi} \int d^2z \{ \mathcal{P}_a \bar{\Pi}^a + d_\alpha \bar{\Pi}^\alpha - \Pi^A \bar{\Pi}^B B_{BA} + \bar{\Pi}^A A_A^I J_I + w_\alpha \bar{\nabla} \lambda^\alpha + \bar{b} \bar{\partial} \bar{c} \} + S_C, \quad (1.3)$$

$$\begin{aligned} \mathcal{H} = & -\frac{1}{2} \mathcal{P}_a \mathcal{P}^a - \frac{1}{2} \Pi^a \Pi_a + d_\alpha \Pi^\alpha + w_\alpha \nabla \lambda^\alpha - \bar{b} \partial \bar{c} - \partial(\bar{b} \bar{c}) + T_C \\ & - \Pi^A A_A^I J_I - d_\alpha W^{\alpha I} J_I - \lambda^\alpha w_\beta U_\alpha^{\beta I} J_I. \end{aligned} \quad (1.4)$$

The vielbein appears through  $\Pi^A = \partial Z^M E_M^A$ , mapping the curved superspace coordinates  $Z^M$ , to the generalized superspace invariants with flat (super) indices  $A$ . The Lorentz connection  $\Omega_{AB}^C$ , enters the covariant derivative  $\nabla$ . The super Kalb-Ramond field is denoted by  $B_{AB}$ , while  $A_A^I$ ,  $W^{\alpha I}$  and  $U_\alpha^{\beta I}$  represent the super Yang-Mills background. All the worldsheet fields above will be detailedly introduced in section 2.

By performing a classical analysis and computing the generalized Poisson brackets associated to  $S$ , we will show that classical nilpotency of the BRST charge (1.1) imposes some constraints on the torsion  $T_{AB}^C$ , the 3-form field strength  $H_{ABC}$ , the curvature tensor  $R_{ABC}^D$ , and the super Yang-Mills field strength  $F_{AB}^I$ , given by

$$\lambda^\alpha \lambda^\beta T_{\alpha\beta}^A = \lambda^\alpha \lambda^\beta H_{A\alpha\beta} = \lambda^\alpha \lambda^\beta \lambda^\gamma R_{\alpha\beta\gamma}^\delta = \lambda^\alpha \lambda^\beta F_{\alpha\beta}^I = 0, \quad (1.5a)$$

in addition to the so-called holomorphicity constraints<sup>1</sup>

$$T_{\alpha\alpha}^\beta = T_{\alpha(ab)} = T_{\alpha\beta b} - H_{\alpha\beta b} = H_{ab\alpha} = \lambda^\alpha \lambda^\beta R_{\alpha\beta}^\gamma = 0, \quad (1.5b)$$

and

$$F_{\alpha a}^I = T_{\alpha\beta a} W^{\beta I}, \quad (1.5c)$$

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<sup>1</sup>This name can be misleading here, as the infinite tension limit is described by the chiral action (1.3) and holomorphicity of the BRST current is trivial.

$$\nabla_\alpha W^{\beta I} - T_{\alpha\gamma}{}^\beta W^{\gamma I} = U_\alpha{}^{\beta I}, \quad (1.5d)$$

$$F_{\alpha\beta}^I = \frac{1}{2} W^{\gamma I} H_{\alpha\beta\gamma}, \quad (1.5e)$$

$$\lambda^\alpha \lambda^\beta \nabla_\alpha U_\beta{}^{\gamma I} = -\lambda^\alpha \lambda^\beta R_{\delta\alpha\beta}{}^\gamma W^{\delta I}. \quad (1.5f)$$

All together, the constraints in (1.5) imply the supergravity and super Yang-Mills equations of motion of the heterotic background, as explained in [7].

This work is organized as follows. Section 2 presents the sectorized model introduced in [8] for the heterotic infinite tension string. Starting with a brief review of Berkovits' original proposal, we will show how the BRST charge was modified to make the sector description manifest and determine the classical conditions for its nilpotency. In section 3, we will discuss the coupling to the heterotic background. For pedagogical reasons, we will analyze first the pure supergravity coupling and extend the results including super Yang-Mills next, explaining in detail how the known background constraints are obtained in the classical analysis. Section 4 discusses the particularities of the sectorized approach and presents some future directions to follow. The reader is advised to go through the appendix A first, as the superspace conventions used here are compactly listed there. Appendix B contains perhaps the simplest worldsheet model for the gauge sector with  $SO(32)$  group and provides some of the ingredients used in the main body of the text.

## 2 The free heterotic string with infinite tension

The heterotic pure spinor string is described in the  $\alpha' \rightarrow 0$  limit by the chiral action

$$S = \frac{1}{2\pi} \int d^2z \{ P_a \bar{\partial} X^a + p_\alpha \bar{\partial} \theta^\alpha + w_\alpha \bar{\partial} \lambda^\alpha + \bar{b} \bar{\partial} \bar{c} \} + S_C. \quad (2.1)$$

$X^a$  and  $\theta^\alpha$  are the  $\mathcal{N} = 1$  superspace coordinates with conjugate momenta  $P_a$  and  $p_\alpha$ , with  $a = 0, \dots, 9$  and  $\alpha = 1, \dots, 16$  denoting the flat vector and spinor indices respectively. The ghost sector is represented by the usual reparametrization ghosts,  $\bar{b}$  and  $\bar{c}$ , the pure spinor  $\lambda^\alpha$ , satisfying  $(\lambda \gamma^a \lambda) = 0$ , and its conjugate  $w_\alpha$ . The gamma matrices satisfy  $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$ , where  $\eta^{ab}$  is the  $SO(1,9)$  metric. The gauge sector is encoded in  $S_C$ . Note that  $S$  has no conformal anomaly and its energy-momentum tensor is given by

$$T = -P_a \partial X^a - p_\alpha \partial \theta^\alpha - w_\alpha \partial \lambda^\alpha - \bar{b} \partial \bar{c} - \partial(\bar{b} \bar{c}) + T_C, \quad (2.2)$$

where  $T_C$  is the gauge sector energy-momentum tensor with central charge  $c = 16$ .

In [3], the action (2.1) was provided with the BRST charge

$$Q = \oint \{ \lambda^\alpha [p_\alpha - \frac{1}{2}(\gamma^a \theta)_\alpha P_a] + \bar{c}T - \bar{b}\bar{c}\partial\bar{c} \}. \quad (2.3)$$

However, it does not correctly describe the expected massless heterotic spectrum, in particular it fails to reproduce the gauge transformations of the supergravity states, which are directly related to the invariance of the theory under general coordinate transformations.

Following the ideas of [6], an alternative BRST charge was proposed in [8] by one of the authors. We will review this construction now.

## 2.1 Review: sectorization and BRST cohomology

Perhaps the first observation hinting at the inadequacy of the BRST charge (2.3) is the existence of an extra nilpotent symmetry of the action (2.1), also linear in  $\lambda^\alpha$ , generated by

$$\mathcal{K} = \oint (\lambda \gamma_a \theta) [\partial X^a + \frac{1}{2}(\theta \gamma^a \partial \theta)]. \quad (2.4)$$

To consistently absorb  $\mathcal{K}$  in the BRST charge, the supersymmetry charges have to be redefined to

$$q_\alpha \equiv \oint \{ p_\alpha + \frac{1}{2}(P_a - \partial X_a)(\gamma^a \theta)_\alpha - \frac{1}{12}(\theta \gamma_a \partial \theta)(\gamma^a \theta)_\alpha \}, \quad (2.5)$$

which in turn brings forth the new invariants:

$$\Pi^a = \partial X^a + \frac{1}{2}(\theta \gamma^a \partial \theta), \quad (2.6a)$$

$$\mathcal{P}_a \equiv P_a - \frac{1}{2}(\theta \gamma_a \partial \theta), \quad (2.6b)$$

$$d_\alpha \equiv p_\alpha - \frac{1}{2}P_a(\gamma^a \theta)_\alpha + \frac{1}{2}\Pi^a(\gamma_a \theta)_\alpha. \quad (2.6c)$$

Note that the operators  $P_a^\pm$  of [8] would be written here as  $P_a^\pm = \mathcal{P}_a \pm \Pi_a$ . The action and its energy-momentum tensor can be expressed in terms of the above invariants as

$$S = \frac{1}{2\pi} \int d^2 z \{ \mathcal{P}_a \bar{\Pi}^a + d_\alpha \bar{\partial} \theta^\alpha + w_\alpha \bar{\partial} \lambda^\alpha + \bar{b} \bar{\partial} \bar{c} \} + S_C \\ - \frac{1}{4\pi} \int d^2 z \{ \Pi^a (\theta \gamma_a \bar{\partial} \theta) - \bar{\Pi}^a (\theta \gamma_a \partial \theta) \}, \quad (2.7)$$

$$T = -\mathcal{P}_a \Pi^a - d_\alpha \partial \theta^\alpha - w_\alpha \partial \lambda^\alpha - \bar{b} \partial \bar{c} - \partial(\bar{b} \bar{c}) + T_C. \quad (2.8)$$

Although not manifestly,  $S$  is invariant under supersymmetry. Consider a transformation with constant parameter  $\xi^\alpha$ , then

$$\begin{aligned}\delta S &= \frac{1}{4\pi} \int d^2z \{ \bar{\Pi}^a (\xi \gamma_a \partial \theta) - \Pi^a (\xi \gamma_a \bar{\partial} \theta) \}. \\ &= \frac{1}{4\pi} \int d^2z \{ (\xi \gamma_a \theta) [\bar{\partial} \Pi^a - \partial \bar{\Pi}^a] \} \\ &= \frac{1}{2\pi} \int d^2z \{ (\xi \gamma_a \theta) (\bar{\partial} \theta \gamma^a \partial \theta) \},\end{aligned}\tag{2.9}$$

Using the property  $(\gamma_{\alpha\beta}^a \gamma_{\gamma\lambda}^b + \gamma_{\alpha\gamma}^a \gamma_{\beta\lambda}^b + \gamma_{\alpha\lambda}^a \gamma_{\beta\gamma}^b) \eta_{ab} = 0$ , the integrand in the last line can be rewritten as

$$(\xi \gamma_a \theta) (\bar{\partial} \theta \gamma^a \partial \theta) = \frac{1}{3} \bar{\partial} [(\xi \gamma_a \theta) (\theta \gamma^a \partial \theta)] - \frac{1}{3} \partial [(\xi \gamma_a \theta) (\theta \gamma^a \bar{\partial} \theta)],\tag{2.10}$$

which proves the invariance of the action  $S$  up to boundary terms.

We will also define the operator

$$\mathcal{H} \equiv -\frac{1}{2} \mathcal{P}_a \mathcal{P}^a - \frac{1}{2} \Pi_a \Pi^a + d_\alpha \partial \theta^\alpha + w_\alpha \partial \lambda^\alpha - \bar{b} \partial \bar{c} - \partial (\bar{b} \bar{c}) + T_C,\tag{2.11}$$

which is the heterotic analogous of the generalized particle-like Hamiltonian for the type II case of [6]. Using these operators, it was shown in [8] that the chiral action  $S$  can be interpreted in terms of two sectors (+) and (−) with characteristic energy-momentum-like tensors

$$T_\pm \equiv \frac{1}{2} (T \pm \mathcal{H}),\tag{2.12}$$

such that

$$T_+ = -\frac{1}{4} \eta^{ab} (\mathcal{P}_a + \Pi_a) (\mathcal{P}_b + \Pi_b) - \bar{b} \partial \bar{c} - \partial (\bar{b} \bar{c}) + T_C,\tag{2.13a}$$

$$T_- = \frac{1}{4} \eta^{ab} (\mathcal{P}_a - \Pi_a) (\mathcal{P}_b - \Pi_b) - d_\alpha \partial \theta^\alpha - w_\alpha \partial \lambda^\alpha.\tag{2.13b}$$

The new BRST charge makes the sectorization of the theory explicit and is given by

$$Q = Q_\lambda + Q_+,\tag{2.14}$$

with

$$Q_\lambda \equiv \oint \lambda^\alpha d_\alpha,\tag{2.15a}$$

$$Q_+ \equiv \oint \{ \bar{c} T_+ - \bar{b} \bar{c} \partial \bar{c} \}.\tag{2.15b}$$

$Q_\lambda$  is very similar to the usual (left-moving) pure spinor BRST charge while  $Q_+$  is composed by the familiar BRST charge coming from the reparametrization symmetry plus an analogous contribution with the operator  $\mathcal{H}$ , *cf.* equation (2.12).

The massless spectrum of the heterotic string consists of non-abelian super Yang-Mills and  $\mathcal{N} = 1$  supergravity, respectively described by the vertex operators

$$U_{SYM} = \lambda^\alpha \bar{c} A_\alpha^I J_I, \quad (2.16a)$$

$$U_{SG} = \lambda^\alpha \bar{c} A_\alpha^a (\mathcal{P}_a + \Pi_a), \quad (2.16b)$$

where  $J_I$  corresponds to (holomorphic) generators of the  $SO(32)$  or  $E(8) \times E(8)$  current algebra, with  $I$  denoting the adjoint representation of the gauge group. BRST-closedness of  $U_{SYM}$  and  $U_{SG}$  with respect to (2.14) provides the known superfield equations of motion at the linearized level,

$$\gamma_{abcde}^{\alpha\beta} D_\alpha A_\beta^I = 0, \quad (2.17a)$$

$$\gamma_{abcde}^{\alpha\beta} D_\beta A_\alpha^f = 0, \quad (2.17b)$$

$$\partial^b \partial_b A_\alpha^a - \partial^a \partial_b A_\alpha^b = 0. \quad (2.17c)$$

The gauge transformations of the superfields, given by

$$\delta_\Sigma A_\alpha^I = D_\alpha \Sigma^I, \quad (2.18a)$$

$$\delta_\Sigma A_\alpha^a = D_\alpha \Sigma^a + \partial^a \Sigma_\alpha, \quad (2.18b)$$

can be written in terms of BRST-exact expressions, as expected. More details can be found in [8].

Next, we will discuss the classical equations associated to the nilpotency of the BRST-charge (2.14) to establish the basis for the curved background analysis of section 3.

## 2.2 Classical analysis

In order to determine the classical conditions to be imposed on the background, it might be useful to understand their meaning in the flat case. Recall that the heterotic action can be cast as

$$S = \frac{1}{2\pi} \int d^2 z \{ \mathcal{P}_a \bar{\Pi}^a + d_\alpha \bar{\partial} \theta^\alpha + w_\alpha \bar{\partial} \lambda^\alpha + \bar{b} \bar{\partial} \bar{c} \} + S_C$$

$$-\frac{1}{4\pi} \int d^2z \{ \Pi^a(\theta\gamma_a\bar{\partial}\theta) - \bar{\Pi}^a(\theta\gamma_a\partial\theta) \}, \quad (2.19)$$

with  $\mathcal{P}_a$  and  $d_\alpha$  being supersymmetric invariants defined in terms of the conjugate momenta of  $X^a$  and  $\theta^\alpha$  respectively, *cf.* equation (2.6). It is convenient, however, to treat them as independent variables. The above action is just one step behind the curved space one that we will define in the next section.

The BRST symmetry is described by the charge displayed in (2.14). To compute the classical BRST transformations of the worldsheet variables, we will rewrite  $Q$  in terms of the fields  $\{X^a, \theta^\alpha, \lambda^\alpha, \bar{c}\}$ , collectively denoted by  $\phi$ , and their canonical conjugates, which are given in terms of  $\{\mathcal{P}_a, d_\alpha, w_\alpha, \bar{b}\}$ . The latter will be denoted by  $\hat{P}_\phi$  and are usually defined with respect to  $\tau$ , the worldsheet time. We will use the Minkowski parametrization with  $z = \sigma - \tau$  and  $\bar{z} = \sigma + \tau$ , where  $\sigma \in [0, 2\pi)$  denotes the spatial coordinate. The derivatives can then be cast as

$$\partial = \frac{1}{2}(\partial_\sigma - \partial_\tau), \quad \bar{\partial} = \frac{1}{2}(\partial_\sigma + \partial_\tau). \quad (2.20)$$

With this convention, the canonical momenta will be defined to be

$$\hat{P}[\phi] \equiv 2\pi \left( \frac{\delta S}{\delta(\bar{\partial}\phi)} - \frac{\delta S}{\delta(\partial\phi)} \right), \quad (2.21)$$

leading to the following identifications:

$$\begin{aligned} \hat{P}[X^a] &= \mathcal{P}_a + \frac{1}{2}(\theta\gamma_a\partial_\sigma\theta), \\ &\equiv \hat{P}_a \end{aligned} \quad (2.22a)$$

$$\begin{aligned} \hat{P}[\theta^\alpha] &= -d_\alpha - \frac{1}{2}(\mathcal{P}_a - \partial_\sigma X_a)(\gamma^a\theta)_\alpha, \\ &\equiv \hat{P}_\alpha \end{aligned} \quad (2.22b)$$

$$\hat{P}[\lambda^\alpha] = w_\alpha, \quad (2.22c)$$

$$\hat{P}[\bar{c}] = -\bar{b}. \quad (2.22d)$$

The fundamental Poisson brackets are simply given by

$$\left\{ \hat{P}[\phi'(\sigma')], \phi(\sigma) \right\}_{P.B.} = -\delta_{\phi, \phi'} \delta(\sigma - \sigma'). \quad (2.23)$$

Therefore, the BRST transformations of the worldsheet fields are easily computed when the



BRST charge is written in terms of  $\hat{P}$  and  $\phi$ . For example,  $Q_\lambda$  in (2.15) is expressed as

$$Q_\lambda = \oint d\sigma \lambda^\alpha \left[ -\hat{P}_\alpha - \frac{1}{2} \hat{P}_a (\gamma^a \theta)_\alpha + \frac{1}{2} \Pi_\sigma^a (\gamma_a \theta)_\alpha \right]. \quad (2.24)$$

Concerning the nilpotency of the BRST charge  $Q$ , it can be stated as

$$Q_\lambda^2 + \{Q_\lambda, Q_+\} + Q_+^2 = 0. \quad (2.25)$$

Because  $Q_\lambda$  is independent of the reparametrization ghosts, each term in the equation above should vanish separately. Therefore, following the classical construction just presented, it is easy to demonstrate that  $Q$  is nilpotent if and only if

$$Q_\lambda^2 = 0, \quad (2.26a)$$

$$\{\lambda^\alpha d_\alpha(\sigma'), T_+(\sigma)\}_{P.B.} = 0, \quad (2.26b)$$

$$\{T_+(\sigma'), T_+(\sigma)\}_{P.B.} = 2T_+ \partial_\sigma \delta(\sigma' - \sigma) + \partial_\sigma T_+ \delta(\sigma' - \sigma). \quad (2.26c)$$

In flat space, it is straightforward to see that all these relations are satisfied. In the next section they will be our guidelines for nontrivial backgrounds. The difference then will be how the background manifests itself in the definition of the conjugate momenta, in particular (2.22a) and (2.22b), which contain the fundamental ingredients of the BRST charge,  $\mathcal{P}_a$  and  $d_\alpha$ .

### 3 Classical consistency of the heterotic background

In this section we will show how the nilpotency conditions discussed above ultimately impose constraints on the heterotic background, providing the expected supergravity and super Yang-Mills equations of motion in superspace detailedly presented in [7] for the pure spinor superstring.

After understanding how the infinite tension string couples to the heterotic background, we will be able to build the operator set necessary for our analysis. The supergravity sector is presented alone beforehand for two reasons. First, to the best of our knowledge, there is no good description for  $\mathcal{N} = 1$  (heterotic) supergravity in any ambitwistor string so far. So this will be a good test for the modifications discussed in [8] for the sectorized string. Second, the generalization from flat space is straightforward and it will help establish the curved superspace language that is extensively used. Next, we will turn on the super Yang-Mills background and extend the results.

### 3.1 Supergravity background and constraints

The curved superspace generalization of (2.7) is given by

$$S = \frac{1}{2\pi} \int d^2z \{ \mathcal{P}_a \bar{\Pi}^a + d_\alpha \bar{\Pi}^\alpha - \Pi^A \bar{\Pi}^B B_{BA} + w_\alpha \bar{\nabla} \lambda^\alpha + \bar{b} \bar{\partial} \bar{c} \} + S_C. \quad (3.1)$$

The vielbein  $E_M^A$ , and the Lorentz connection  $\Omega_{AB}^C$ , enter the action through the generalized superspace invariants and the covariant derivative<sup>2</sup>, given by

$$\bar{\Pi}^A = \bar{\partial} Z^M E_M^A, \quad (3.2a)$$

$$\bar{\nabla} \lambda^\alpha = \bar{\partial} \lambda^\alpha + \lambda^\beta \bar{\Pi}^A \Omega_{A\beta}^\alpha, \quad (3.2b)$$

and analogous expressions for  $\Pi^A$  and  $\nabla \lambda^\alpha$ , where  $Z^M$  denotes the curved  $\mathcal{N} = 1$  superspace coordinates  $X^m$  and  $\theta^\mu$ . The curved vector and spinor indices are being respectively denoted by  $m = 0, \dots, 9$  and  $\mu = 1, \dots, 16$ . Notice that in this language the supermetric  $G_{MN}$  is written in terms of the flat metric as  $G_{MN} = E_M^a E_N^b \eta_{ab}$ . The coupling with the Kalb-Ramond superfield can be easily written with explicit curved space indices,

$$\begin{aligned} S_B &= -\frac{1}{2\pi} \int d^2z \{ \Pi^A \bar{\Pi}^B B_{BA} \} \\ &= -\frac{1}{2\pi} \int d^2z \{ \partial Z^M \bar{\partial} Z^N B_{NM} \}, \end{aligned} \quad (3.3)$$

with

$$B_{AB} = (-1)^{A(B+N)} E_B^N E_A^M B_{MN}. \quad (3.4)$$

This form is more suitable to show the gauge invariance of the action with respect to the transformations  $\delta B_{MN} = \partial_M \Sigma_N - (-1)^{MN} \partial_N \Sigma_M$ . More details on the conventions used here can be found in Appendix A. Concerning the dilaton superfield, it plays no role in the classical description and this can be seen from the fact that its coupling to the action naively vanishes in the  $\alpha' \rightarrow 0$  limit.

Following the analysis of subsection 2.2,  $\mathcal{P}_a$  and  $d_\alpha$  can be viewed as independent objects invariant under supersymmetry, and the flat space limit of  $S$  is recovered when we express them in terms of regular variables, *cf.* (2.6), together with the non-vanishing components of  $E$  and  $B$

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<sup>2</sup>Due to the pure spinor constraint the action has a gauge symmetry with parameter  $\varphi_a$  of the form  $\delta_\varphi w_\alpha = \varphi_a (\gamma^a \lambda)_\alpha$ . Therefore, to work only with gauge invariant quantities we must impose  $\lambda^\alpha (\gamma^a \lambda)_\beta \Omega_{A\alpha}^\beta = 0$ .

in that limit:

$$\begin{aligned} E_m^a &= \delta_m^a, & E_\mu^a &= -\frac{1}{2}(\gamma^a \theta)_\mu \\ E_\mu^\alpha &= \delta_\mu^\alpha, & B_{m\mu} &= -B_{\mu m} = \frac{1}{2}E_m^a(\gamma_a \theta)_\mu. \end{aligned} \quad (3.5)$$

The energy-momentum tensor of the curved space action is given by

$$T = -\mathcal{P}_a \Pi^a - d_\alpha \Pi^\alpha - w_\alpha \nabla \lambda^\alpha - \bar{b} \partial \bar{c} - \partial(\bar{b} \bar{c}) + T_C, \quad (3.6)$$

and the curved version of  $\mathcal{H}$  in (2.11) is simply

$$\mathcal{H} = -\frac{1}{2} \mathcal{P}_a \mathcal{P}^a - \frac{1}{2} \Pi^a \Pi_a + d_\alpha \Pi^\alpha + w_\alpha \nabla \lambda^\alpha - \bar{b} \partial \bar{c} - \partial(\bar{b} \bar{c}) + T_C. \quad (3.7)$$

The BRST-charge in the curved background has the same structure of (2.14) and the presence of the background can be seen through the canonical conjugates of the superspace coordinates  $Z^M$ , denoted by  $\hat{P}_M$ . Using the definition (2.21), one obtains

$$\hat{P}_M = E_M^a \mathcal{P}_a - E_M^\alpha d_\alpha + \partial_\sigma Z^N B_{NM} + w_\alpha \lambda^\beta \Omega_{M\beta}^\alpha, \quad (3.8)$$

which enables us to rewrite  $\mathcal{P}_a$  and  $d_\alpha$  as

$$d_\alpha = -E_\alpha^M \hat{P}_M + (\Pi^A + \bar{\Pi}^A) B_{A\alpha} + w_\gamma \lambda^\beta \Omega_{\alpha\beta}^\gamma, \quad (3.9a)$$

$$\mathcal{P}_a = E_a^M \hat{P}_M - (\Pi^A + \bar{\Pi}^A) B_{Aa} - w_\gamma \lambda^\beta \Omega_{a\beta}^\gamma. \quad (3.9b)$$

To go from (3.8) to (3.9), we have used the inverse vielbein  $E_A^M$ , such that  $E_A^M E_M^B = \delta_A^B$  and  $E_M^A E_A^N = \delta_M^N$ .

For the BRST-charge to be nilpotent, and thus well-defined as such, the background superfields need to satisfy a number of constraints. To find these constraints, we begin by computing the transformations of the worldsheet fields under the action of  $Q_\lambda$ . Using the graded Poisson brackets

$$\{\hat{P}_M(\sigma'), Z^N(\sigma)\}_{P.B.} = -\delta_M^N \delta(\sigma - \sigma'), \quad (3.10)$$

$$\{w_\alpha(\sigma'), \lambda^\beta(\sigma)\}_{P.B.} = -\delta_\alpha^\beta \delta(\sigma - \sigma'), \quad (3.11)$$

we obtain the following transformations<sup>3</sup>

$$\delta \lambda^\alpha = -\lambda^\beta \Lambda_\beta^\alpha, \quad (3.12a)$$

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<sup>3</sup>For a reasonably detailed exposition of similar calculations, see [9].

$$\delta w_\alpha = \Lambda_\alpha^\beta w_\beta + \epsilon d_\alpha, \quad (3.12b)$$

$$\delta \Pi^a = -\Pi^b \Lambda_b^a - \epsilon \lambda^\alpha \Pi^A T_{A\alpha}^a, \quad (3.12c)$$

$$\delta \Pi^\alpha = -\Pi^\beta \Lambda_\beta^\alpha + \epsilon \nabla \lambda^\alpha - \epsilon \lambda^\beta \Pi^A T_{A\beta}^\alpha, \quad (3.12d)$$

$$\delta \mathcal{P}_a = \Lambda_a^b \mathcal{P}_b - \epsilon \lambda^\beta T_{\beta a}^b \mathcal{P}_b + \epsilon \lambda^\gamma T_{\gamma a}^\beta d_\beta + \epsilon \lambda^\beta \Pi^A H_{A\beta a} - \epsilon \lambda^\gamma \lambda^\beta w_\delta R_{\beta a \gamma}^\delta, \quad (3.12e)$$

$$\delta d_\alpha = \Lambda_\alpha^\beta d_\beta + \epsilon \lambda^\beta T_{\beta \alpha}^b \mathcal{P}_b - \epsilon \lambda^\gamma T_{\gamma \alpha}^\beta d_\beta - \epsilon \lambda^\beta \Pi^A H_{A\beta \alpha} + \epsilon \lambda^\gamma \lambda^\beta w_\delta R_{\beta \alpha \gamma}^\delta, \quad (3.12f)$$

$$\delta \Omega_\alpha^\beta = \nabla \Lambda_\alpha^\beta - \epsilon \lambda^\gamma \Pi^A R_{A\gamma \alpha}^\beta. \quad (3.12g)$$

Here  $\epsilon$  is a constant anticommuting parameter and we have defined

$$\Lambda_A^B \equiv \epsilon \lambda^\alpha \Omega_{\alpha A}^B, \quad \Omega_\alpha^\beta \equiv \Pi^A \Omega_{A\alpha}^\beta. \quad (3.13)$$

Now we can compute the transformation of  $\lambda^\alpha d_\alpha$ , whose vanishing is equivalent to the first condition displayed in (2.26):

$$\delta(\lambda^\alpha d_\alpha) = \epsilon \lambda^\alpha \lambda^\beta [T_{\alpha\beta}^a \mathcal{P}_a - T_{\alpha\beta}^\gamma d_\gamma - \Pi^A H_{A\alpha\beta} + w_\delta \lambda^\gamma R_{\alpha\beta\gamma}^\delta]. \quad (3.14)$$

Hence the first set of constraints required for the nilpotency of  $Q$  is:

$$\lambda^\alpha \lambda^\beta T_{\alpha\beta}^A = \lambda^\alpha \lambda^\beta H_{A\alpha\beta} = \lambda^\alpha \lambda^\beta \lambda^\gamma R_{\alpha\beta\gamma}^\delta = 0. \quad (3.15)$$

Nilpotency of the BRST charge also requires  $\delta T_+$  to vanish. This is just another way of stating the condition (2.26b). The operator  $T_+$  is obtained from the definition (2.12) and the curved versions of  $T$  and  $\mathcal{H}$ , respectively (3.6) and (3.7). It can be cast as

$$T_+ = -\frac{1}{4} \eta^{ab} (\mathcal{P}_a + \Pi_a) (\mathcal{P}_b + \Pi_b) - \bar{b} \partial \bar{c} - \partial(\bar{b} \bar{c}) + T_C. \quad (3.16)$$

Now, to compute  $\delta T_+$  we just have to use the transformations of  $\mathcal{P}_a$  and  $\Pi^a$  in (3.12) and the result is

$$\begin{aligned} \delta T_+ = & -\frac{1}{2} \epsilon (\mathcal{P}^a + \Pi^a) \lambda^\alpha d_\beta T_{\alpha a}^\beta + \frac{1}{2} \epsilon (\mathcal{P}^a \mathcal{P}^b - \Pi^a \Pi^b) \lambda^\alpha T_{\alpha ab} - \frac{1}{2} \epsilon \lambda^\alpha \mathcal{P}^a \Pi^b H_{\alpha ab} \\ & + \frac{1}{2} \epsilon (\mathcal{P}^a + \Pi^a) \lambda^\alpha \Pi^\beta (T_{\alpha\beta a} - H_{\alpha\beta a}) + \frac{1}{2} \epsilon (\mathcal{P}^a + \Pi^a) \lambda^\alpha \lambda^\beta w_\gamma R_{\beta a \alpha}^\gamma. \end{aligned} \quad (3.17)$$

For this expression to vanish, we need to impose another set constraints:

$$T_{\alpha a}^\beta = T_{\alpha(ab)} = T_{\alpha\beta b} - H_{\alpha\beta b} = H_{ab\alpha} = \lambda^\alpha \lambda^\beta R_{\alpha a \beta}^\gamma = 0. \quad (3.18)$$

In the usual pure spinor superstring, this set comes from the holomorphicity of the BRST charge [7].

Finally, rewriting  $T_+$  in terms of the canonical conjugates and using the Poisson brackets of (3.10) together with

$$\{\bar{b}(\sigma'), \bar{c}(\sigma)\}_{P.B.} = \delta(\sigma' - \sigma), \quad (3.19)$$

$$\{T_C(\sigma'), T_C(\sigma)\}_{P.B.} = 2T_C \partial_\sigma \delta(\sigma' - \sigma) + \partial_\sigma T_C \delta(\sigma' - \sigma), \quad (3.20)$$

we can show that

$$\{T_+(\sigma'), T_+(\sigma)\}_{P.B.} = 2T_+ \partial_\sigma \delta(\sigma' - \sigma) + \partial_\sigma T_+ \delta(\sigma' - \sigma). \quad (3.21)$$

Therefore, the three conditions of (2.26) for classical nilpotency of the BRST charge in a supergravity background are all satisfied provided the constraints displayed in (3.15) and (3.18).

### 3.2 Turning on the super Yang-Mills background

In order to find the remaining heterotic background constraints, we need to consider the case in which the super Yang-Mills fields are present. We will introduce the minimal coupling between the gauge potential  $A_M^I$  and the currents  $J_I$ , such that the action has the form

$$S = \frac{1}{2\pi} \int d^2z \{ \mathcal{P}_a \bar{\Pi}^a + d_\alpha \bar{\Pi}^\alpha - \Pi^A \bar{\Pi}^B B_{BA} + \bar{A}^I J_I + w_\alpha \bar{\nabla} \lambda^\alpha + \bar{b} \bar{\partial} \bar{c} \} + S_C, \quad (3.22)$$

where  $\bar{A}^I \equiv \bar{\partial} Z^M A_M^I$ . Note this is equivalent to replacing  $\mathcal{P}_a \rightarrow \mathcal{P}_a + A_a^I J_I$  and  $d_\alpha \rightarrow d_\alpha - A_\alpha^I J_I$  in the action 3.1, with  $A_A^I = E_A^M A_M^I$ .

The gauge invariance of the action (3.22) with respect to the super Yang-Mills background is straightforward to demonstrate. Consider the gauge transformations with superparameter  $\Sigma^I$ ,

$$\delta_\Sigma A_M^I = \partial_M \Sigma^I + [A_M, \Sigma]^I, \quad (3.23)$$

where

$$[A_M, \Sigma]^I = f_{JK}^I A_M^J \Sigma^K, \quad (3.24)$$

and  $f_{JK}^I$  denotes the structure constants of the gauge group. Substituting the transformation

(3.23) in the variation of the action and integrating by parts, one obtains

$$\delta_\Sigma S = -\frac{1}{2\pi} \int d^2z (\bar{\partial} J_I + f_{IJ}{}^K \bar{A}^J J_K) \Sigma^I. \quad (3.25)$$

The expression inside the parentheses is just the equation of motion for the current  $J_I$  in the presence of the super Yang-Mills source and thus vanishes at the classical level. More details on this construction can be found in appendix B where an explicit realization of the gauge sector is given for the group  $SO(32)$  in terms of worldsheet fermions. For convenience, we can introduce the super field strength of  $A_M^I$ , given by

$$F_{MN}^I = \partial_M A_N^I - (-1)^{MN} \partial_N A_M^I + f_{JK}{}^I A_M^J A_N^K, \quad (3.26)$$

which transforms covariantly under (3.23),

$$\delta_\Sigma F_{MN}^I = [F_{MN}, \Sigma]^I. \quad (3.27)$$

The coupling to the super Yang-Mills background changes the energy-momentum tensor to

$$T = -\mathcal{P}_a \Pi^a - d_\alpha \Pi^\alpha - w_\alpha \nabla \lambda^\alpha - \bar{b} \partial \bar{c} - \partial(\bar{b} \bar{c}) + T_C - \Pi^A A_A^I J_I, \quad (3.28)$$

and we also expect the operator  $\mathcal{H}$  to be modified accordingly. It was suggested in [6] that fluctuations of the background would be manifested through  $\mathcal{H}$ . Therefore, inspired by the superstring integrated vertex, we propose

$$\begin{aligned} \mathcal{H} = & -\frac{1}{2} \mathcal{P}_a \mathcal{P}^a - \frac{1}{2} \Pi^a \Pi_a + d_\alpha \Pi^\alpha + w_\alpha \nabla \lambda^\alpha - \bar{b} \partial \bar{c} - \partial(\bar{b} \bar{c}) + T_C \\ & - \Pi^A A_A^I J_I - d_\alpha W^{\alpha I} J_I - \lambda^\alpha w_\beta U_\alpha{}^{\beta I} J_I, \end{aligned} \quad (3.29)$$

where  $W^{\alpha I}$  and  $U_\alpha{}^{\beta I}$  are background superfields<sup>4</sup> which will be related to (3.26). Again, using (2.12),  $T_+$  can be cast as

$$\begin{aligned} T_+ = & -\frac{1}{4} \eta^{ab} (\mathcal{P}_a + \Pi_a) (\mathcal{P}_b + \Pi_b) - \bar{b} \partial \bar{c} - \partial(\bar{b} \bar{c}) + T_C \\ & - \Pi^A A_A^I J_I - \frac{1}{2} d_\alpha W^{\alpha I} J_I - \frac{1}{2} \lambda^\alpha w_\beta U_\alpha{}^{\beta I} J_I, \end{aligned} \quad (3.30)$$

and we now have all the ingredients to analyze the BRST symmetry in this background.

The modification of the action entails a change in the classical BRST transformations of the

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<sup>4</sup>As before, we must impose  $\lambda^\alpha (\gamma^a \lambda)_\beta U_\alpha{}^{\beta I} = 0$  in order to respect the gauge invariance implied by the pure spinor constraint.

worldsheet fields. This is clearly seen from the canonical conjugates of the superspace coordinates, which now have a linear dependence on the gauge field:

$$\hat{P}_M = E_M^a \mathcal{P}_a - E_M^\alpha d_\alpha + \partial_\sigma Z^N B_{NM} + w_\alpha \lambda^\beta \Omega_{M\beta}^\alpha + A_M^I J_I. \quad (3.31)$$

To compute these transformations we will use the fundamental brackets of (3.10), (3.11), (3.19) and (3.20), together with

$$\{T_C(\sigma'), J_I(\sigma)\}_{P.B.} = J_I \partial_\sigma \delta(\sigma - \sigma') + \partial_\sigma J_I \delta(\sigma' - \sigma), \quad (3.32a)$$

$$\{J_I(\sigma'), J_J(\sigma)\}_{P.B.} = f_{IJ}^K J_K \delta(\sigma' - \sigma), \quad (3.32b)$$

which are derived in Appendix B.

Considering first  $Q_\lambda$ , it is clear that most of the transformations displayed in (3.12) remain unchanged, except for  $\delta\mathcal{P}_a$  and  $\delta d_\alpha$ , which are now given by

$$\begin{aligned} \delta\mathcal{P}_a &= \Lambda_a^b \mathcal{P}_b - \epsilon \lambda^\beta T_{\beta a}^b \mathcal{P}_b + \epsilon \lambda^\gamma T_{\gamma a}^\beta d_\beta \\ &\quad + \epsilon \lambda^\alpha \Pi^A H_{A\alpha a} - \epsilon \lambda^\gamma \lambda^\beta w_\delta R_{\beta a \gamma}^\delta - \epsilon \lambda^\beta F_{\beta a}^I J_I, \end{aligned} \quad (3.33a)$$

$$\begin{aligned} \delta d_\alpha &= \Lambda_\alpha^\beta d_\beta + \epsilon \lambda^\beta T_{\beta \alpha}^b \mathcal{P}_b - \epsilon \lambda^\gamma T_{\gamma \alpha}^\beta d_\beta \\ &\quad - \epsilon \lambda^\beta \Pi^A H_{A\beta \alpha} + \epsilon \lambda^\gamma \lambda^\beta w_\delta R_{\beta \alpha \gamma}^\delta + \epsilon \lambda^\beta F_{\beta \alpha}^I J_I, \end{aligned} \quad (3.33b)$$

where

$$F_{AB}^I = (-1)^{A(B+N)} E_B^N E_A^M F_{MN}^I. \quad (3.34)$$

We can now easily compute the transformation of  $\lambda^\alpha d_\alpha$  and the result is

$$\delta(\lambda^\alpha d_\alpha) = \epsilon \lambda^\alpha \lambda^\beta [T_{\alpha\beta}^a \mathcal{P}_a - T_{\alpha\beta}^\gamma d_\gamma - \Pi^A H_{A\alpha\beta} + w_\delta \lambda^\gamma R_{\alpha\beta\gamma}^\delta + F_{\alpha\beta}^I J_I]. \quad (3.35)$$

Thus, together with the constraints displayed in (3.15) we also need to impose

$$\lambda^\alpha \lambda^\beta F_{\alpha\beta}^I = 0, \quad (3.36)$$

in order to satisfy the first nilpotency condition, *cf.* equation (2.26a).

Next, to compute  $\delta T_+$  and evaluate the condition (2.26b) it is worth noting that  $T_C$  and  $J_I$  now have nonvanishing transformations with respect to  $Q_\lambda$ , given by

$$\delta T_C = J_I \partial \Sigma^I, \quad (3.37a)$$

$$\delta J_I = -f_{IJ}^K \Sigma^J J_K, \quad (3.37b)$$

where we have defined the gauge-like parameter  $\Sigma^I \equiv \epsilon \lambda^\alpha A_\alpha^I$ . The introduction of  $\Sigma^I$  is convenient when we look at the variations of the background superfields,

$$\delta A^I = \nabla \Sigma^I - \epsilon \lambda^\alpha \Pi^A F_{A\alpha}^I, \quad (3.38a)$$

$$\delta W^{\alpha I} = -W^{\beta I} \Lambda_\beta^\alpha - [\Sigma, W^\alpha]^I + \epsilon \lambda^\beta \nabla_\beta W^{\alpha I}, \quad (3.38b)$$

$$\delta U_\alpha^{\beta I} = \Lambda_\alpha^\gamma U_\gamma^{\beta I} - U_\alpha^\gamma \Lambda_\gamma^\beta - [\Sigma, U_\alpha^\beta]^I + \epsilon \lambda^\gamma \nabla_\gamma U_\alpha^{\beta I}, \quad (3.38c)$$

which can be interpreted in terms of a gauge-like transformation with parameter  $\Sigma$ , a Lorentz-like transformation with parameter  $\Lambda$ , and a superspace translation, with  $\nabla_\alpha$  denoting the covariant derivative with respect to the local symmetries, *e.g.*

$$\nabla_\beta W^{\alpha I} = D_\beta W^{\alpha I} + \Omega_{\beta\gamma}^\alpha W^{\gamma I} + [A_\beta, W^\alpha]^I. \quad (3.39)$$

Gathering all these results and using the supergravity constraints of (3.18), we obtain

$$\begin{aligned} \delta T_+ &= \frac{1}{2} \epsilon \lambda^\alpha (\mathcal{P}^a - \Pi^a) [F_{\alpha a}^I - T_{\alpha\beta a} W^{\beta I}] J_I - \frac{1}{2} \epsilon \lambda^\alpha \lambda^\beta w_\gamma [\nabla_\alpha U_\beta^{\gamma I} + R_{\delta\alpha\beta}^\gamma W^{\delta I}] J_I \\ &\quad + \epsilon \lambda^\alpha \Pi^\beta [F_{\alpha\beta}^I + \frac{1}{2} H_{\alpha\beta\gamma} W^{\gamma I}] J_I + \frac{1}{2} \epsilon \lambda^\alpha d_\beta [\nabla_\alpha W^{\beta I} - T_{\alpha\gamma}^\beta W^{\gamma I} - U_\alpha^{\beta I}] J_I \\ &\quad - \frac{1}{2} \epsilon \lambda^\alpha F_{\alpha\beta}^I W^{\beta J} J_I J_J, \end{aligned} \quad (3.40)$$

Hence, we have to further impose the following constraints:

$$F_{\alpha a}^I = T_{\alpha\beta a} W^{\beta I}, \quad (3.41a)$$

$$\nabla_\alpha W^{\beta I} - T_{\alpha\gamma}^\beta W^{\gamma I} = U_\alpha^{\beta I}, \quad (3.41b)$$

$$F_{\alpha\beta}^I = \frac{1}{2} W^{\gamma I} H_{\alpha\beta\gamma}, \quad (3.41c)$$

$$\lambda^\alpha \lambda^\beta \nabla_\alpha U_\beta^{\gamma I} = -\lambda^\alpha \lambda^\beta R_{\delta\alpha\beta}^\gamma W^{\delta I}. \quad (3.41d)$$

Note that the last line of (3.40) vanishes automatically after the identification in (3.41c).

As a final consistency check, it is not difficult to show that  $T_+$  satisfies

$$\{T_+(\sigma'), T_+(\sigma)\}_{P.B.} = 2T_+ \partial_\sigma \delta(\sigma' - \sigma) + \partial_\sigma T_+ \delta(\sigma' - \sigma), \quad (3.42)$$

which demonstrates the last necessary condition for the nilpotency of the BRST charge at the classical level.



## 4 Discussion

It is possible to show that the constraints displayed in (3.15), (3.18), (3.36) and (3.41) imply the ten-dimensional supergravity and super Yang-Mills equations of motion. Instead of presenting these results, which for the pure spinor superstring were originally obtained and detailedly studied in [7], we will discuss the particularities of the infinite tension string model.

As presented in subsection 2.2, there are in principle three independent conditions to check in order to ensure classical nilpotency of the BRST charge,

$$Q_\lambda^2 = 0, \quad (4.1a)$$

$$\{\lambda^\alpha d_\alpha(\sigma'), T_+(\sigma)\}_{P.B.} = 0, \quad (4.1b)$$

$$\{T_+(\sigma'), T_+(\sigma)\}_{P.B.} = 2T_+ \partial_\sigma \delta(\sigma' - \sigma) + \partial_\sigma T_+ \delta(\sigma' - \sigma). \quad (4.1c)$$

The first one is identical to the condition on the left-moving BRST charge of the usual pure spinor superstring and not surprisingly provides the so-called nilpotency constraints, given by

$$\lambda^\alpha \lambda^\beta T_{\alpha\beta}{}^A = \lambda^\alpha \lambda^\beta H_{A\alpha\beta} = \lambda^\alpha \lambda^\beta \lambda^\gamma R_{\alpha\beta\gamma}{}^\delta = \lambda^\alpha \lambda^\beta F_{\alpha\beta}^I = 0, \quad (4.2)$$

exactly as in [7]. The second condition can be stated as

$$[Q_\lambda, T_+] = 0, \quad (4.3)$$

and leads to the constraints of (3.18) and (3.41), which were obtained in [7] by requiring holomorphicity of the BRST current. In the present model, holomorphicity plays no role when it comes to imposing constraints. This is so because the heterotic background coupled action, given by

$$S = \frac{1}{2\pi} \int d^2z \{ \mathcal{P}_a \bar{\Pi}^a + d_\alpha \bar{\Pi}^\alpha - \Pi^A \bar{\Pi}^B B_{BA} + \bar{A}^I J_I + w_\alpha \bar{\nabla} \lambda^\alpha + \bar{b} \bar{\partial} \bar{c} \} + S_C, \quad (4.4)$$

is still chiral. Therefore the remaining constraints should manifest themselves through the condition (4.3). To interpret it, it might be useful to recall that conformal symmetry is preserved at the classical level, such that  $[Q_\lambda, T] = 0$ . We are then left with

$$[Q_\lambda, \mathcal{H}] = 0, \quad (4.5)$$

*cf.* the definition 2.12. This is precisely the *ad hoc* condition used in [6] for the type II construction. Here, however, it is naturally embedded in the BRST operator. From the sectorized point

of view, the condition above is equivalent to the conservation of the BRST charge separately in each sector.

Concerning the third condition in (4.1), it was verified to hold *independently* of the background constraints. This is partially connected to the classical conformal symmetry but we do not have a clear understanding so far. It implies, for example, that

$$\{\mathcal{H}(\sigma'), \mathcal{H}(\sigma)\}_{P.B.} = 2T \partial_\sigma \delta(\sigma' - \sigma) + \partial_\sigma T \delta(\sigma' - \sigma). \quad (4.6)$$

We expect this relation to hold in the type II case as well.

An interesting observation is that the background can be absorbed by a field redefinition in the action, such that

$$S = \frac{1}{2\pi} \int d^2z \{ \bar{\partial} Z^M \mathbf{P}_M + w_\alpha \bar{\partial} \lambda^\alpha + \bar{b} \bar{\partial} \bar{c} \} + S_C, \quad (4.7)$$

where

$$\begin{aligned} \mathbf{P}_M \equiv & E_M^a (\mathcal{P}_a + \Pi^A B_{Aa} + A_a^I J_I + w_\gamma \lambda^\beta \Omega_{a\beta}^\gamma) \\ & - E_M^\alpha (d_\alpha - \Pi^A B_{A\alpha} - A_\alpha^I J_I - w_\gamma \lambda^\beta \Omega_{\alpha\beta}^\gamma). \end{aligned} \quad (4.8)$$

In this case, one can work with  $\mathbf{P}_M$  as a fundamental field and rewrite the BRST charge by expressing  $d_\alpha$  and  $\mathcal{P}_a$  as functions of  $\mathbf{P}_M$  and the other worldsheet fields. Therefore we have instead a free action and the heterotic background appears as a deformation of the BRST charge, supporting the observation made by Chandia and Vallilo that the vertex operators in this model could be seen as fluctuations of  $\mathcal{H}$ . Needless to emphasize, quantum consistency of the theory would be much easier to verify in this approach, similarly to what was done in [5] for the NS-NS background. As in there, we expect the dilaton superfield to start playing a fundamental role in the quantum formulation of the theory.

It should be noted that in the original ambitwistor strings, either in RNS or with pure spinors, the heterotic supergravity sector has some unsolved issues. For example, Mason and Skinner [2] computed the n-particle tree level amplitude and could not interpret them in terms of standard space-time gravity, with the 3-point amplitude suggesting a (Weyl)<sup>3</sup>-type vertex. On the other hand, the supergravity vertex of [8] in the sectorized string seems to provide the correct OPE structure in the 3-point amplitude, resembling the usual heterotic pure spinor string up to numerical factors. This subject deserves a deeper investigation and might shed some light on the model. Naturally, if we go to 4-point amplitudes or higher we need also the integrated vertices. We still do not have a simple proposal for such operators. However, there are interesting hints

pointing out that the holomorphic sectorization can be extended the bosonic string and to the Ramond-Neveu-Schwarz and Green-Schwarz formalisms [10]. We hope that understanding this construction will provide a better basis to approach the problem of the integrated vertex operator in the infinite tension limit using pure spinors. Once this step is taken, we will finally be able to compute the tree level amplitudes to compare them with the Cachazo-He-Yuan formulae [1] and investigate the modular invariance of the theory at 1-loop, for example, as done in [11].

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## A Superspace notation and conventions

In this work, we use the following sets of indices:

$$\begin{aligned}
a, b, \dots = 0 \text{ to } 9 : & \quad \text{ten-dimensional tangent space vector,} \\
\alpha, \beta, \dots = 1 \text{ to } 16 : & \quad \text{ten-dimensional tangent space chiral spinor,} \\
m, n, \dots = 0 \text{ to } 9 : & \quad \text{ten-dimensional manifold vector,} \\
\mu, \nu, \dots = 1 \text{ to } 16 : & \quad \text{ten-dimensional manifold chiral spinor,} \\
A, B, \dots & \quad \text{collectively denote } (a, \alpha), (b, \beta), \dots, \\
M, N, \dots & \quad \text{collectively denote } (m, \mu), (n, \nu), \dots.
\end{aligned}$$

Our conventions concerning differential forms are the same as those in [12]. In particular, given a manifold with vielbein  $E$  and connection  $\Omega$ , we define the torsion to be

$$T = dE + E \wedge \Omega, \tag{A.1}$$

or, in components,

$$T_{NM}{}^A = 2\partial_{[N}E_{M]}{}^A + (-1)^{N(B+M)}E_M{}^B\Omega_{NB}{}^A - (-1)^{MB}E_N{}^B\Omega_{MB}{}^A, \tag{A.2}$$

where the graded symmetrization is defined as

$$\Xi_{[NM]} \equiv \frac{1}{2} \left[ \Xi_{NM} - (-1)^{NM} \Xi_{MN} \right], \tag{A.3}$$

and the indices appearing at the exponents should be replaced by their gradings, *i.e.* +1 for spinorial indices and 0 otherwise. As usual, one can write the torsion in terms of tangent space

indices by contracting it with vielbeins:

$$T_{BC}{}^A = (-1)^{B(C+M)} E_C{}^M E_B{}^N T_{NM}{}^A. \quad (\text{A.4})$$

Another important quantity is the curvature tensor, defined in terms of the connection as

$$R = d\Omega + \Omega \wedge \Omega, \quad (\text{A.5})$$

or, in components,

$$R_{NMA}{}^B = 2\partial_{[N}\Omega_{M)A}{}^B + (-1)^{N(A+C+M)}\Omega_{MA}{}^C\Omega_{NC}{}^B - (-1)^{M(A+C)}\Omega_{NA}{}^C\Omega_{MC}{}^B. \quad (\text{A.6})$$

Finally, the 3-form  $H = dB$  is given in components by  $H_{MNP} = 3\partial_{[M}B_{NP]}$ .

## B $SO(32)$ realization of the gauge sector

Here we will present a realization of the action  $S_C$  describing the gauge sector of the heterotic string, focusing on the  $SO(32)$  group, which has a simpler construction.

Concerning notation, the vector and the adjoint representations of the group, will be respectively denoted by the indices  $i, j, k, \dots = 1, \dots, 32$  and  $I, J, K, \dots = 1, \dots, 496$ . The metric set  $(\delta^{ij}, \delta^{IJ}, \delta_{ij}, \delta_{IJ})$  will be used to raise and to lower the group indices.

The generators of the  $SO(32)$  group will be denoted by the anti-Hermitian operators  $T^I$ . The algebra can be cast as

$$[T_I, T_J] = f_{IJ}{}^K T_K, \quad (\text{B.1})$$

where  $f_{IJK} = f_{IJ}{}^L \delta_{LK}$  are real and totally antisymmetric structure constants constrained to satisfy the Jacobi identity

$$f_{IJ}{}^M f_{MK}{}^L + f_{JK}{}^M f_{MI}{}^L + f_{KI}{}^M f_{MJ}{}^L = 0. \quad (\text{B.2})$$

The action  $S_C$  consists of a (free) set of 32 real worldsheet fermions,  $\psi_i$ , such that

$$S_C = \frac{1}{4\pi} \int d^2z \psi^i \bar{\partial} \psi^j \delta_{ij}. \quad (\text{B.3})$$

The associated energy-momentum tensor is

$$T_C = -\frac{1}{2} \psi^i \partial \psi^j \delta_{ij}. \quad (\text{B.4})$$

Using the simple OPE

$$\psi^i(z)\psi^j(y) \sim \frac{\delta^{ij}}{(z-y)}, \quad (\text{B.5})$$

one can easily compute

$$T_C(z)T_C(y) \sim \frac{(\frac{16}{2})}{(z-y)^4} + \frac{2T_C}{(z-y)^2} + \frac{\partial T_C}{(z-y)}, \quad (\text{B.6})$$

showing that the central charge of the system is 16, as required by the vanishing conformal anomaly in the heterotic string.

The  $SO(32)$  group structure in the worldsheet theory can be seen through the current

$$J_I \equiv \frac{1}{2}(T_I^{jk}\psi_j\psi_k). \quad (\text{B.7})$$

Observe that  $J^I$  is conserved,  $\bar{\partial}J^I = 0$ , which follows from the classical equation of motion for  $\psi_i$ . For completeness, we can mention that the currents  $J_I$  define an Affine Lie algebra at the quantum level, which can be read from the OPE:

$$J_I(z)J_J(y) \sim \frac{1}{2} \frac{\text{Tr}(T_I T_J)}{(z-y)^2} + f_{IJ}^K \frac{J_K}{(z-y)}. \quad (\text{B.8})$$

Following the notation of subsection 2.2, we can see that the canonical conjugate of  $\psi^i$  is identified with  $\psi^i$  itself, as usual for fermionic systems. Therefore we have to use Dirac's procedure to deal with this constraint in order to obtain the Dirac brackets for  $\psi^i$ , given by

$$\{\psi^i(\sigma'), \psi^j(\sigma)\} = \delta^{ij}\delta(\sigma - \sigma'), \quad (\text{B.9})$$

and show that the currents satisfy

$$\{T_C(\sigma'), J_I(\sigma)\} = J_I \partial_\sigma \delta(\sigma - \sigma') + \partial_\sigma J_I \delta(\sigma' - \sigma), \quad (\text{B.10})$$

$$\{J_I(\sigma'), J_J(\sigma)\} = f_{IJ}^K J_K \delta(\sigma' - \sigma), \quad (\text{B.11})$$

which are used in section 3.

The simplest interacting model involving the currents  $J_I$  is the minimal coupling to an external source  $\bar{A}_I$ , such that

$$S_C^{int} = S_C + \frac{1}{2\pi} \int d^2z J_I \bar{A}^I. \quad (\text{B.12})$$

In this case, the equation of motion for the current can be determined to be

$$\bar{\partial}J_I + f_{IJ}{}^K \bar{A}^J J_K = 0. \quad (\text{B.13})$$

It is interesting to observe that this coupling has a very natural symmetry at the classical level. Consider the transformation

$$\delta \bar{A}^I = \bar{\partial} \Sigma^I - f_{JK}{}^I \Sigma^J \bar{A}^K, \quad (\text{B.14})$$

where  $\Sigma^I$  is a generic parameter in the adjoint representation of  $SO(32)$ . It is straightforward to show that  $S_C^{int}$  transforms as

$$\delta S_C^{int} = -\frac{1}{2\pi} \int d^2z (\bar{\partial}J_I + f_{IJ}{}^K \bar{A}^J J_K) \Sigma^I, \quad (\text{B.15})$$

which vanishes for classical configurations of  $J_I$ , *cf.* equation (B.13).

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